

**Mathematics A**

**T064**

**Wednesday, 30/10/2013**

**8:30 – 11:30 AM**

WORKFORCE DEVELOPMENT AUTHORITY



P.O. BOX 2707 Kigali, Rwanda Tel: (+250) 255113365

**ADVANCED LEVEL NATIONAL EXAMINATION, 2013,  
TECHNICAL AND PROFESSIONAL TRADES**

**EXAM TITLE: Mathematics A**

**OPTIONS:** - Computer Science (CSC)  
- Computer Science and Management (CSM)

**DURATION: 3hours**

**INSTRUCTIONS:**

The paper contains **two** sections :

Section I: Sixteen **(16)** Compulsory questions

**55marks**

Section II: Five **(5)** questions – Choose any **three (3)**.

**45marks**

**SECTION I: Attempt all the 16 questions 55marks**

**01.** Solve the equations:

i.  $|2x - 5| = 3$ .

**4marks**

ii.  $\sin x = 2\sin\left(\frac{x}{2}\right)$ .

**02.** The cubic polynomial  $6x^3 + 7x^2 + ax + b$  has a remainder of 72 when divided by  $(x - 2)$  and is exactly divisible by  $(x + 1)$ . Calculate a and b.

**3marks**

**03.** Find the oblique asymptotes of the curve  $y = -1 + \frac{2}{3}\sqrt{x^2 - 4x - 5}$ .

**4marks**

**04.** Prove that the following function is even  $f(x) = \ln\left(\frac{3^x}{9^x + 1}\right)$ .

**3marks**

**05.** a) A, B and C are the angles of a triangle such that  $\cos A = \frac{3}{5}$  and

$\cos B = \frac{5}{13}$ . Without using tables or calculator find the value of

(i)  $\tan 2A$  ; (ii)  $\cos(A + B)$ ; (iii)  $\cos C$ .

**4marks**

b) Prove the identity  $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$ .

**06.** Given that  $2 + i$  is a root of the equation  $z^3 - 11z + 20 = 0$ , find the remaining roots.

**4marks**

**07.** Given that  $y = e^{\tan x}$ , show that  $\frac{d^2y}{dx^2} - (2\tan x + \sec^2 x) \frac{dy}{dx} = 0$

**3marks**

**08.** a) Determine the points on the curve  $2y = (3 - x^2)$  at which the tangent is parallel to  $x + y = 0$ .

**2marks**

b) Find the equation of tangent and normal to the curve  $y = x^2 + 4x + 1$  at the point whose abscissa is  $x = 3$ .

**2marks**

**09.** Given  $A = \begin{bmatrix} 3 & 5 \\ 4 & 2 \end{bmatrix}$ , Calculate  $A^2 - 5A - 11I$  and hence find  $A^{-1}$ .

**3marks**

**10.** Given that  $\frac{p}{2x+3} + \frac{q}{3x+2} = \frac{1}{(2x+3)(3x+2)}$ , find the values of the constants p and q. Hence

calculate  $\int \frac{dx}{(2x+3)(3x+2)}$ .

**4marks**

**11.** If  $\left(\frac{3}{2} + \frac{\sqrt{3}}{2}i\right)^{50} = 3^{25}(x + iy)$ , then find  $(x, y)$ .

**4marks**

**12.** If  $\alpha$  and  $\beta$  are the roots of the equation  $2x^2 + 3x + 5 = 0$ , find the value of the determinant

$$\begin{vmatrix} 0 & \beta & \beta \\ \alpha & 0 & \alpha \\ \beta & \alpha & 0 \end{vmatrix}.$$

**3marks**

**13.** Find the point of intersection of lines

$$\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1} \quad \text{and} \quad \frac{x+3}{-36} = \frac{y-3}{2} = \frac{z-6}{4}.$$

**3marks**

14. Given the polynomial  $P(x) = 2x^3 - 13x^2 - 10x + 21$ .
- a. Determine three real numbers  $a, b$  and  $c$  such that  
 $P(x) = (x - 1)(ax^2 + bx + c)$ . **2marks**
- b. Then solve in  $IR$  the equation  $P(x) = 0$ . **1.5marks**
- c. Deduce the solutions in  $IR$  of the equation  
 $2(\ln x)^3 + 13(\ln x)^2 - 10(\ln x) + 21 = 0$  **1.5marks**
15. Evaluate  $\int_0^2 |x^2 + 2x - 3| dx$ . **4marks**

**SECTION II: Attempt any three questions 45marks**

16. a) If  $x = \frac{t}{1+t}$  and  $y = \frac{t^3}{1+t}$  find  $\frac{dy}{dx}$  and hence find the tangent at  $t = 1$ . **5marks**
- b) Given the function  $f(x) = \frac{5x+7}{3x+2}$ ,
- i) Find inverse of  $f(x)$ . **3marks**
- ii) Calculate  $(f^{-1} \circ f)(x)$ . **2marks**
- c) Find equation of straight line perpendicular to both the lines  
 $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z+2}{3}$  and  $\frac{x+2}{2} = \frac{y-5}{-1} = \frac{z+3}{2}$  and passing through the point  $(2,3,1)$ . **5marks**
17. a) Given that  $\log_2 x + 2 \log_4 y = 4$ ; show that  $xy = 16$ . Hence solve for  $x$  and  $y$  the simultaneous equations:  

$$\begin{cases} \log_{10}(x+y) = 1 \\ \log_2 x + 2 \log_4 y = 4. \end{cases}$$
 **4marks**
- b) Use De Moivre's theorem to show that  
 $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ . **4marks**
- c) Calculate the following limits
- i)  $\lim_{x \rightarrow 0} (1 - 2x)^{3/x}$ . **3.5marks**
- ii)  $\lim_{x \rightarrow +\infty} \frac{5^{x+1} + 7^{x+1}}{5^x + 7^x}$ . **3.5marks**
18. Given the function  $f(x) = \sqrt{x^2 - 1}$
- a. Find the domain of definition. **2marks**
- b. Is  $f(x)$  odd or even function? **1mark**
- c. Find asymptotes to the curve. **3marks**
- d. Compute the first derivative and study its sign. **2marks**
- e. Compute the second derivative and study its sign. **2marks**
- f. Find the variation table. **2marks**
- g. Sketch the graph of the curve in Cartesian plane. **3marks**

19. a) Sketch the parabola  $y = 2x - x^2$  and the line  $y = -x$  in Cartesian plane. **5marks**
- b) Find the area bounded by the curve  $y = 2x - x^2$  and the straight line  $y = -x$ . **5marks**
- c) Solve the simultaneous equation  $\begin{cases} x + y = 3e \\ \ln x + \ln y = 2 + \ln 2 \end{cases}$ . **5marks**

20. a) The table below shows the weight of students who underwent medical examination at the university hospital.

Weight (in kg)	Number of students
40-44	3
45-49	10
50-54	15
55-59	10
60-64	4
65-69	5
70-74	4
75-79	6
80-84	1

Calculate:

- i. The mode. **3marks**
- ii. The median and mean weight of the students **4marks**
- iii. Draw a cumulative frequency graph. **3marks**
- b) The correlation coefficient between the variable X and Y is  $r = 0.60$ . If  $\sigma_x = 1.50$  and  $\sigma_y = 2$ ,  $\bar{x} = 10$ ,  $\bar{y} = 20$ ; find the equation of regression line
- i. Y on X. **2.5marks**
- ii. X on Y. **2.5marks**